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***Bentley New Village***

**Progression of**

**Calculations**

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**Calculation Policy**

**Aims:**

At Bentley New Primary we aim to develop pupils’ calculation skills through the provision of practical, oral, mental and written mathematical activities. Pupils will learn how to use a range of models and images to support their mental and informal written methods of calculation, leading to the development of efficient written methods as the children move through school. Teachers will present their classes with opportunities to develop their understanding of all 4 operations using the concrete, pictorial and abstract.

The 2014 National Curriculum provides a structured and systematic approach to the teaching of calculation. The aim is for mental calculations and written procedures to be performed efficiently, fluently, and accurately with understanding. Procedures and understanding are to be developed in tandem. End of key stage expectations are explicit in the programme of study. At Bentley New Village Primary School, we have a consistent approach to the teaching of written calculation methods in order to ensure continuity and progression across the school.

**Age related expectations and Mastery:**

Although the National Curriculum gives guidance as to the expectations for the teaching of written methods, **it is vital that pupils are taught according to the stage that they are currently working at,** being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

At the centre of the mastery approach to the teaching of mathematics is the belief that all pupils have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly with calculation strategies, pupils must not simply rote-learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. We focus on all children achieving what is expected of their age group and not going beyond this. Evidence shows that children need to be able to understand a concept, apply it in a range of situations and then be creative to really understand it. Simply going beyond their age group does not guarantee they understand something, it just means they have heard it.

Please note that the principle of the concrete-pictorial-abstract (CPA) approach is that for pupils to have a true understanding of a mathematical concept, they need to master all three phases. Reinforcement is achieved by going back and forth between these representations. For example, if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, ‘proving’ something or ‘working out’ could involve use of the concrete or pictorial. In short, these are not always ‘exclusive’ representations.

At our school no child will be taught content from the year group above them, they will spend time becoming true masters of content, applying and being creative with new knowledge and skills in multiple ways.

**Purpose:**

This policy makes teachers aware of the written strategies that children are formally taught as they progress through school. The policy only details the strategies - teachers must plan opportunities for children to apply these; for example, when solving problems, developing reasoning skills or where opportunities emerge elsewhere in the curriculum. **Teachers should provide a context for calculation.**  It is important that any type of calculation is given a real life context or problem solving approach to help build children’s understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods. It is also important for children to be confident to use mental and written strategies to explain their thinking**.** This must be a priority within calculation lessons**.** Written methods need to be viewed as tools to enable children to solve problems and record their thinking in an organised way. This guide is not prescriptive and is based on suggested calculations from the national curriculum (2014). The following methods are organised by year group though should be used flexibly according to individual need and ability.

**Before carrying out a calculation, children will be encouraged to consider:**

* Can I do it in my head? (using rounding, adjustment) Secure mental methods which are developed from early years
* The size of an approximate answer (estimation). Pupils are expected to use their developing number sense from Year 1 to make predictions about the answers to their calculations. As their range of mental strategies increases, these predictions and, later, estimates should become increasingly sophisticated and accurate. All teaching of calculation should emphasise the importance of making and using these estimates to check, first, the sense and, later, the accuracy of their calculations.
* Could I use jottings to keep track of the calculation? Visual models and images including number lines and arrays and move on to experience of expanded methods to develop understanding and avoid rote-learning
* Which resources to use to support their calculation. The development of an efficient written calculation relies on practical hands-on experience including a range of manipulatives

**Glossary**

**2-digit** – a number with 2 digits like 23, 45, 12 or 60

**3-digit** – a number with 3 digits like 123, 542, 903 or 561

**Addition facts** – knowing that 1+1 = 2 and 1+3 = 4 and 2+5 = 7. Normally we only talk about number facts with totals of 20 and under.

**Array** - An array is an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.

**Bridge to ten** – a strategy when using numberlines. Adding a number that takes you to the next ‘tens’ number.

**Bus Stop Method** - traditional method for division with a single digit divisor

**Concrete apparatus** – objects to help children count – these are most often cubes (multilink) but can be anything they can hold and move. Dienes Base 10 (purple hundreds, tens and units blocks), Numicon, Cuisenaire rods are also referred to as concrete apparatus.

**Column chunking** – method of division involving taking chunks or groups of the divisor away from the larger number

**Decimal** **number** – a number with a decimal point

**Divisor** – the smaller number in a division calculation. The number in each group for chunking.

**Double** – multiply a number by 2

**Exchanging** – Moving a ‘ten’ or a ‘hundred’ from its column into the next column and splitting it up into ten ‘ones’ (or ‘units’) or ten ‘tens’ and putting it into a different column

**Expanded Multiplication** – a method for multiplication where each stage is written down and then added up at the end in a column

**Find the difference** – A method for subtraction involving counting up from the smaller to the larger number

**Grid method** – a method for multiplying two numbers together involving partitioning

**Half** - a number, shape or quantity divided into 2 equal parts

**Halve** – divide a number by 2

**Integer** - a number with no decimal point

**Inverse** – the opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division

**Jigsaw numbers** – addition facts to 100, see number bonds to 100

**Long Multiplication** – column multiplication where only the significant figures are noted

**Number bonds to ten** – 2 numbers that add together to make ten, like 2 and 8, or 6 and 4.

**Number bonds to 100** – 2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88

**Numberline** – a line either with numbers or without (a blank numberline). Children use this tool to help them count on for addition or subtraction and also in multiplication and division.

**Numberline Chunking** - method of division involving taking chunks or groups of the divisor away from the larger number

**Number sentence** – writing out a calculation with just the numbers in a line e.g. 2+4=6 or 35 ÷7 = 5 or 12 x 3 =36 or 32 – 5 = 27

**Partition** – split up a larger number into the hundreds, tens and units. e.g. 342 – 300 and 40 and 2

**Place Value** – knowing that in the number 342 – the ‘3’ means ‘3 hundreds’, the ‘4’ means ‘4 tens’ and the ‘2’ means ‘2’

**Quarter** - a number, shape or quantity divided into 4 equal parts

**Recombine** – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add the hundreds together, then add the tens to that total, and then add the units to that total

**Remainder** – a whole number left over after a division calculation

**Repeated addition** – repeatedly adding groups of the same size for multiplication

**Significant digit** – the digit in a number with the largest value. eg. in 34 – the most significant digit is the 3, as it has a value of ‘30’ and the ‘4’ only has a value of ‘4’

**Single digit** – a number with only one digit. These are always less than 10.

**Taking away** – a method for subtraction involving counting backwards from the larger to the smaller number

**Tens number** - a number in the ten times tables – 10,20,30,40, 50, etc.

**Unit** – another term for single digit numbers. The right hand column in column methods is the ‘units’ column. This should be replaced by ones

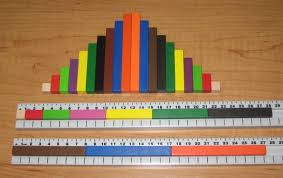
**Resources**

**Dienes/Base 10**

Dienes, although it has been used in schools for years is a crucial step in knowing what a ‘one’ (unit), a ten, a hundred and a thousand look like and how they can be added together and split up to form smaller and larger numbers.

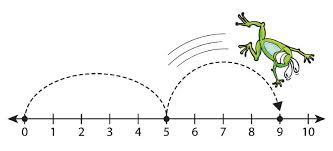
**Numicon**

Numicon is an especially useful resource as it can be used for teaching all four operations as well as fractions, decimals, percentages and a range of other aspects of maths. Each piece represents an integer from 1 to 10. The children love using it as it is colourful and tactile



**Cuisenaire Rods and a number track**

Although these little rods (that represent integers from 1 to 10) can be used for a range of aspects of maths, we normally use them for multiplication and division. They are also really useful for addition.



**Numberlines**

Numberlines are a mainstay of teaching calculations. We have pre numbered and blank numberlines in school that children can write on, or they can draw their own as appropriate for the calculation



[](http://www.google.co.uk/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=0ahUKEwigibWR0tDNAhUFKsAKHbXaDLcQjRwIBw&url=http://www.creativeclassrooms.co.nz/red-yellow-counters.html&bvm=bv.126130881,d.ZGg&psig=AFQjCNG-YSkY_TIarciYYJ0f2YrmlNNYEQ&ust=1467406712909914)**Cubes/Counters**

Cubes and counters are a very versatile piece of maths equipment that can support many areas of the maths curriculum.

**Times tables -** The NC encourages a focussed learning of times tables. This should be used throughout and children should know multiplication facts for all tables up to 10 x 10 by the end of Year 4.

**Monitoring and evaluation**

The teaching of calculations will be monitored using evidence collected from:

* Planning scrutiny
* Classroom observations
* Scrutiny of children’s work
* AFL opportunities within lessons
* Discussions with children
* Evidence sheets

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| **Videos to support mathematical teaching and learning** | |
| **Number and Place value**  https://www.ncetm.org.uk/resources/40534 KS1 ‐ Counting in steps of one and ten  KS1 ‐ Partitioning in different ways KS1 ‐ Addition and Subtraction  KS1 ‐ Using resources to develop fluency and understanding  KS2 ‐ Partitioning (subtraction) | **Multiplication**  https://www.ncetm.org.uk/resources/40530 KS1 ‐ Multiple Representations of Multiplication  KS1‐ The commutative law for multiplication Lower KS2 ‐ Grid multiplication as an interim step  Upper KS2 ‐ Moving from grid to a column |
| **Number facts**  https://www.ncetm.org.uk/ resources/40533  KS1 ‐ Number bonds to ten  KS1 ‐ Consolidation and practice (Addition and Subtraction)  KS1 ‐ Reinforcing Table Facts  KS1 ‐ Rapid recall of multiplication facts | **Algebra**  https://www.ncetm.org.uk/ resources/43649  KS1 ‐ Look at ’missing numbers’ KS2 ‐ Equations and substitution KS3 ‐ Factorising\* |
| **Subtraction**  https://www.ncetm.org.uk/ resources/40532  Lower KS2 – Partitioning  Lower KS2 ‐ Discussing Subtraction Strategies  Lower KS2 ‐ Developing Column Subtraction Upper KS2‐ Column Subtraction | **Fractions**  https://www.ncetm.org.uk/ resources/43609  KS1 ‐ Adding fractions and mixed numbers  KS2 ‐ Using an array to add fractions KS2 ‐ Bar model dividing by fractions KS3 ‐ Fraction wall to add fractions\* |
| **Division**  https://www.ncetm.org.uk/ resources/43589  KS1‐ Sharing and grouping  KS 2 ‐ Place value counters for division  KS 3 ‐ Group working on problems\* | **Multiplicative reasoning**  https://www.ncetm.org.uk/ resources/43669  KS2 ‐ Bar model for multiplication  KS3 ‐ Ratio and proportion\* |

**Mathematical Language**

|  |  |
| --- | --- |
|  |  |
|  |  |
| ones | units |
| is equal to | equals |
| zero | oh (the letter O) |

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning. Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct, for example, using the term ‘regroup’ rather than ‘carry’ or ‘borrow.’

For all calculations we need children to practically carry out the calculation (action), say what they have done (language) and then record this. Therefore, it is extremely important that children are introduced to the mathematical language first and practise using this to explain their actions, thinking and the reasoning behind the strategy they have used.

Before children move on to formal methods, which are outlined in the policy, the recoding should always be children’s own concept to begin with. Children will need to use the mathematical language to explain their workings and understanding.

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**Division**

Share equally

Divide by

Group

Halve, quarter etc

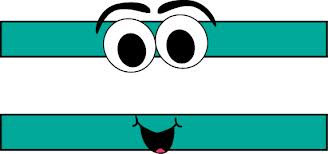
Chunk

Multiple

Pairs

Remainders

Array



**Equals**

Is the value same as

Is equivalent to

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**Multiply**

Sets of

Lots of

Groups of

Times

Multiply by

Double – Twice, three times etc

Product

Multiple

Factor

Repeated addition

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**Subtraction**

Take away

Minus

Decrease

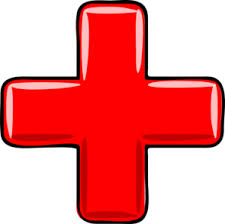
Less than

Difference

How much more

Subtract

Left

** Addition**

Add

And

Plus

Increase

More than

Total

Altogether

Sum of

Children will have been introduced to calculation through a range of counting activities songs, rhymes and games, with the emphasis on practical, hands-on experiences. Children will have been introduced to the terms more and less practically before any number knowledge has been introduced. This is an essential step before teaching calculation.

**Addition**

|  |  |  |  |
| --- | --- | --- | --- |
| Objective and Strategies | Concrete | Pictorial | Abstract |
| Number Bonds |  | [https://s-media-cache-ak0.pinimg.com/236x/a4/11/07/a411079575f7fc4d1b122046baf40638.jpg](https://www.google.co.uk/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=0ahUKEwin2K6fxdDNAhUIIsAKHfmvDGEQjRwIBw&url=https://www.pinterest.com/drnicki7/number-bonds/&psig=AFQjCNGa5inWzvjCRyWQSxsU6bn2gZSrBQ&ust=1467403244819435) | 1 + \_\_\_\_\_ = 10  10 = 9 + 1  How many different ways can you make 10? |
| Combining two parts to make a  whole: part- whole model | Use cubes to add two numbers together as a group or in a bar. | Image result for part whole modelC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\2GUHBRQ0\Simple-Flower-Outline-12183-large[1].png  8  1  Image result for part whole model addition  Use pictures to add two numbers together as a group or in a bar. | 4 + 3 = 7  10= 6 + 4  5  3  Use the part-part whole diagram as shown above to move into the abstract. |
| Starting at the bigger number and counting on | Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. | 12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer. | 5 + 12 = 17  Place the larger number in your head and count on the smaller number to find your answer. |
| Re-grouping to make 10. | 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. | Use pictures or a number line. Regroup or partition the smaller number to make 10. | 7 + 4= 11  If I am at seven, how many more do I need to make 10. How many more do I add on now? |
| Adding three single digits | 4 + 7 + 6= 17  Put 4 and 6 together to make 10. Add on 7.    Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit. | C:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\OZODVY09\pitr-Candy-icon[1].png  Add together three groups of objects. Draw a picture to recombine the groups to make 10.  +  + | Combine the two numbers that make 10 and then add on the remainder. |
| Column method- no re-grouping | 24 + 15=  Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters. | After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.  T O |  |
| Column method- re-grouping | Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.    Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.  This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.  As children move on to decimals, money and decimal place value counters can be used to support learning. | Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. | Start by partitioning the numbers before moving on to clearly show the exchange below the addition.  As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. |
| Other ideas | Concrete experiences using a range of apparatus.  Finding one more using:   * Actual objects * Beginning of a number line (cardinal) * Knowledge of number system   2-3 more, number in head and count on | Use of numberlines to count on in steps of one.  Could be a ‘numbered’ line or ruler. |  |

**Subtraction**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Objective and Strategies | Concrete | Pictorial | | Abstract | |
| Taking away ones | Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4 | Cross out drawn objects to show what has been taken away. | | 18 -3= 15  8 – 2 = 6 | |
| Counting back | Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4  http://3.bp.blogspot.com/-mFqQPE4k1TE/VGzRNnUu30I/AAAAAAAAAJM/12p6qvgkmoE/s1600/EvenOdd_ColoredCounters_Scattered.jpgUse counters and move them away from the group, as you take them away counting backwards as you move them. | Count back on a number line or number track    Start at the bigger number and count back the smaller number showing the jumps on the number line.    This can progress all the way to counting back using two 2 digit numbers. | | Put 13 in your head, count back 4. What number are you at? Use your fingers to help. | |
| Find the difference | Image result for two towers of cubesCompare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference  Use basic bar models with items to find the difference | Count on to find the difference.  http://image.slidesharecdn.com/intro-to-sm-1220840292402057-8/95/intro-to-singapore-math-13-728.jpg?cb=1345557040Draw bars to find the difference between 2 numbers. | | Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. | |
| Part Part Whole Model | Link to addition- use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part?  10 - 6 = | Use a pictorial representation of objects to show the part part whole model. | | 10  5  Move to using numbers within the part whole model. | |
| Make 10 | 14 – 9 =  Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9. | Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. | | 16 – 8=  How many do we take off to reach the next 10?  How many do we have left to take off? | |
| Column method without regrouping | Use Base 10 to make the bigger number then take the smaller number away.  Show how you partition numbers to subtract. Again make the larger number first. | | Draw the Base 10 or place value counters alongside the written calculation to help to show working. | | http://media.showmeapp.com/files/205114/pictures/thumbs/1100814/last_thumb1379615590.jpg  [https://encrypted-tbn3.gstatic.com/images?q=tbn:ANd9GcS1ohiHkzn0cS0nvwRP-5EyK0TDGl_A1tbsAl0XjNPBssTas4YVeQ](http://www.google.co.uk/url?sa=i&rct=j&q=&esrc=s&source=images&cd=&cad=rja&uact=8&ved=0CAcQjRxqFQoTCPyKt_H6h8kCFUNEFAodiFAGCA&url=http://huppiemama.com/teaching-subtraction-using-manipulatives/&bvm=bv.106923889,d.d2s&psig=AFQjCNEr_xOQu7fhwvMOMFTIen6kpdc03g&ust=1447317198959935)This will lead to a clear written column subtraction. |
| Column method with regrouping | Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.  Make the larger number with the place value counters  Start with the ones. Can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.    Now I can subtract my ones.  Now look at the tens. Can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction    Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount. | Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  When confident, children can find their own way to record the exchange/regrouping.  Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup. | | Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method.  This will lead to an understanding of subtracting any number including decimals. | |

**Multiplication**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Objective and Strategies | Concrete | Pictorial | | Abstract | |
| Doubling | Use practical activities to show how to double a number. | Draw pictures to show how to double a number. | | Partition a number and then double each part before recombining it back together. | |
| Counting in multiples | Count in multiples supported by concrete objects in equal groups. | Use a number line or pictures to continue support in counting in multiples. | | Count in multiples of a number aloud.  Write sequences with multiples of numbers.  2, 4, 6, 8, 10  5, 10, 15, 20, 25 , 30 | |
| Repeated addition | Use different objects to add equal groups. |  | | Write addition sentences to describe objects and pictures. | |
| Arrays- showing commutative multiplication | http://www.australiancurriculumlessons.com.au/wp-content/uploads/2013/05/arrays-multiplication-division-lesson.jpgCreate arrays using counters/ cubes to show multiplication sentences. | http://mathcentral.uregina.ca/QQ/database/QQ.02.06/maro1.1.gifDraw arrays in different rotations to find **commutative** multiplication sentences.  Link arrays to area of rectangles. | | Use an array to write multiplication sentences and reinforce repeated addition. | |
| Grid Method | Show the link with arrays to first introduce the grid method.  4 rows of 10  4 rows of 3  Move on to using Base 10 to move towards a more compact method.  4 rows of 13  Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones making any exchanges needed.    Then you have your answer. | Children can represent the work they have done with place value counters in a way that they understand.  http://www.highviewschool.org.uk/wp-content/uploads/2014/05/IMG_0499-300x225.jpgThey can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below. | | Start with multiplying by one digit numbers and showing the clear addition alongside the grid.  http://www.mumsnet.com/system/1/assets/files/000/006/988/6988/35010b289/original/328x164xgrid-method-explained-2.jpg.pagespeed.ic.zL-KyDdiL2.jpg  Moving forward, multiply by a 2 digit number showing the different rows within the grid method. | |
| Column multiplication | Children can continue to be supported by place value counters at the stage of multiplication.  https://primarysite-prod.s3.amazonaws.com/0c4eb252d34643748228179a3d582154_1x1.jpeg  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below. | | *C:\Users\nathan.crook\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Outlook\3IR2FLXR\photo (5).JPG*Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  C:\Users\nathan.crook\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.Outlook\3IR2FLXR\photo (7).JPG | | Start with long multiplication, reminding the children about lining up their numbers clearly in columns.  If it helps, children can write out what they are solving next to their answer.  http://amsi.org.au/teacher_modules/B1/B1t102.png  http://ictedusrv.cumbria.ac.uk/maths/SecMaths/U1/images/pic018.gif  This moves to the more compact method. |

**Division**

|  |  |  |  |
| --- | --- | --- | --- |
| Objective and Strategies | Concrete | Pictorial | Abstract |
| Sharing objects into groups | I have 10 cubes, can you share them equally in 2 groups? | Children use pictures or shapes to share quantities.  C:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].pngC:\Users\b.smith\AppData\Local\Microsoft\Windows\Temporary Internet Files\Content.IE5\C9ORAZE7\Simple-Flower-Outline-12183-large[1].png  8 ÷ 2 = 4 | Share 9 buns between three people.  9 ÷ 3 = 3 |
| Division as grouping | Divide quantities into equal groups.  Use cubes, counters, objects or place value counters to aid understanding. | Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  http://gcamath3.weebly.com/uploads/9/1/4/0/9140392/200455_orig.jpg | 28 ÷ 7 = 4  Divide 28 into 7 groups. How many are in each group? |
| Division within arrays | Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15  15 ÷ 5 = 3 3 x 5 = 15 | Draw an array and use lines to split the array into groups to make multiplication and division sentences. | Find the inverse of multiplication and division sentences by creating four linking number sentences.  7 x 4 = 28  4 x 7 = 28  28 ÷ 7 = 4  28 ÷ 4 = 7 |
| Division with a remainder | 14 ÷ 3 =  Divide objects between groups and see how much is left over  Image result for counters | Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder. | Complete written divisions and show the remainder using r.  http://amsi.org.au/teacher_modules/G7/G7_qt2%202.png |
| Short division | Use place value counters to divide using the bus stop method alongside  42 ÷ 3=  Start with the biggest place value. We are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14. | http://www.studyzone.org/testprep/math4/d/division2.gifStudents can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. | Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately. |

**Fluency, reasoning and problem solving**

What does fluency, reasoning and problem solving look like in solving calculation questions?

These are the three aims from the 2014 Mathematics National Curriculum which are to ensure all pupils:

* become fluent in the fundamentals of mathematics, through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
* reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
* can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

The 2014 mathematics curriculum states that ‘Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas… (all) pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems.’ The BNVP Scheme of work will develop this idea further.

Examples of fluency, reasoning and problem solving:

8 x 5 = 40

Starting with this problem, pupils who demonstrate good fluency, reasoning and problem solving skills are able to use this fact to create others such as:

* 5 x 8 = 40
* 80 x 50 = 4000
* 8 x 5 = 20 x 2
* 40 ÷ 5 = 8
* (2 x 4) x 5 = 10 x 4
* 16 x 2.5 = 40
* 40 ÷ 8 = 5
* 0.8 x 5 = 4
* 8 x 50 = 400
* 5 x 8 = (5 x 10) – (5 x 2)
* 5 x 8 = 10 x 4
* 5 x 8 = 8 + 8 + 8 + 8 + 8
* 40 = 8 x 5